

1-3

Solving Equations With a Variable on Both Sides

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I CAN... write and solve equations with a variable on both sides to solve problems.

VOCABULARY

- identity



Activity



Assess



EXPLORE & REASON

Some friends want to see a movie that is showing at two different theaters in town. They plan to share three tubs of popcorn during the movie.



	Theater A	Theater B
Ticket Price	\$14.50	\$13.00
Popcorn	\$5.75	\$6.75

- Construct Arguments** Which movie theater should the friends choose? Explain.
- For what situation would the total cost at each theater be exactly the same? Explain.
- There are different methods to solving this problem. Which do you think is the best? Why?



ESSENTIAL QUESTION

How do you create equations with a variable on both sides and use them to solve problems?



EXAMPLE 1 Solving Equations With a Variable on Both Sides

A. What is the value of x in the equation shown?

$$3x - 10 + 4x = -2(x - 4) + 9$$

Combine like terms.

$$7x - 10 = -2x + 8 + 9$$

Distribute the -2 .

$$7x + 2x = 8 + 9 + 10$$

Collect like terms on the same side of the equation.

$$9x = 27$$

$$\frac{9x}{9} = \frac{27}{9}$$

$$x = 3$$

B. What is the value of n in the equation shown?

$$\frac{1}{2}(n - 4) - 7 = -2n + 6$$

$$\frac{1}{2}(n - 4) = -2n + 13$$

Multiply each side by 2 to eliminate the fraction.

$$n - 4 = -4n + 26$$

$$n + 4n = 26 + 4$$

Collect like terms on the same side of the equation.

$$5n = 30$$

$$n = 6$$



Try It! 1. Solve each equation.

a. $100(z - 0.2) = -10(5z + 0.8)$ b. $\frac{5}{8}(16d + 24) = 6(d - 1) + 1$





Activity



Assess

**CONCEPTUAL
UNDERSTANDING****EXAMPLE 2****Understand Equations With Infinitely Many or No Solutions****A. What is the value of x in $4x + 6 = 2(2x + 3)$?**Use algebra tiles to represent and solve $4x + 6 = 2(2x + 3)$.

$$4x + 6 = 2(2x + 3)$$

Use 2 groups of $2x + 3$ to represent $2(2x + 3)$.

$$4x - 4x + 6 = 4x - 4x + 6$$

$$6 = 6$$

Because $6 = 6$ is always a true statement and it is equivalent to the original equation, the equation is true for every value of x .

VOCABULARY

Since this equation is true for all values of the variable, it is sometimes referred to as having *infinitely many solutions*.

The equation $4x + 6 = 2(2x + 3)$ is true for all values of x .An equation that is true for all values of the variables is an **identity**.**B. What is the value of x in $6x - 5 = 2(3x + 4)$?**Solve for x .

$$6x - 5 = 2(3x + 4)$$

$$6x - 5 = 6x + 8$$

$$6x - 6x - 5 = 6x - 6x + 8$$

Maintain the equality by subtracting $6x$ from each side.

$$-5 = 8$$

There is no value of x that makes the equation true. Therefore, the equation has no solution.**STUDY TIP**

Recall that you can assign a value to the variable in an equation to check whether the equation is true.

**Try It!****2. Solve each equation. Is the equation an identity? Explain.**

a. $t - 27 = -(27 - t)$

b. $16(4 - 3m) = 96\left(-\frac{m}{2} + 1\right)$

APPLICATION

EXAMPLE 3 Solve Mixture Problems

Arabica coffee costs \$28 per pound and Robusta coffee costs \$8.75 per pound. How many pounds of Arabica coffee must you mix with 3 pounds of Robusta coffee to make a blend that costs \$15.50 per pound?

Organize the information in a table.

	Price (\$/lb)	Amount (lb)	Total cost (\$)
Arabica coffee	28.00	a	$28a$
Robusta coffee	8.75	3	26.25
Coffee blend	15.50	$a + 3$	$15.5(a + 3)$

Write an equation to represent the situation.

$$28a + 26.25 = 15.5(a + 3)$$

$$28a + 26.25 = 15.5a + 46.5$$

$$28a - 15.5a = 46.5 - 26.25$$

$$12.5a = 20.25$$

$$a = 1.62$$

You must mix 1.62 pounds of Arabica coffee with 3 pounds of Robusta coffee to make a blend that costs \$15.50 per pound.

USE APPROPRIATE TOOLS

How are tables helpful in organizing quantities to represent situations algebraically?

- Try It!** 3. How many pounds of Arabica coffee should you mix with 5 pounds of Robusta coffee to make a coffee blend that costs \$12.00 per pound?

APPLICATION

EXAMPLE 4 Use Equations to Solve Problems

Cameron pays \$0.95 per song with his current music service. A new music download service charges \$0.89 per song with a \$12 joining fee. Should Cameron switch to the new service?

Formulate Write an equation to represent when the cost for any number of songs, s , is the same for both services.

New music service = Cameron's current music service

$$0.89s + 12 = 0.95s$$

Compute Solve the equation to find the number of songs at which the cost for each option will be the same.

$$0.89s - 0.89s + 12 = 0.95s - 0.89s$$

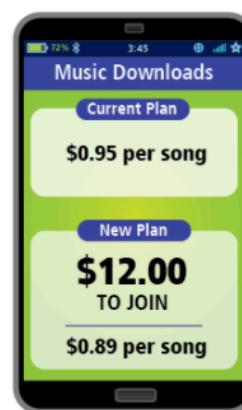
$$12 = 0.06s$$

$$\frac{12}{0.06} = \frac{0.06s}{0.06}$$

$$200 = s$$

Interpret The cost of the two options will be the same for 200 songs.

If Cameron plans to purchase more than 200 songs, he should switch to the new service because it will cost less than his current service.



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Concept
Summary

Assess

EXAMPLE 4 CONTINUED



- Try It!** 4. Cameron's friend tells him of another service that has a \$15 joining fee but charges \$0.80 per song. At what number of songs does this new service become a less expensive option than Cameron's current service?

**CONCEPT SUMMARY** Solve Equations with a Variable on Both Sides

Linear equations can be used to solve mathematical and real-world problems.

WORDS

You can use properties of equality to solve an equation with variables on both sides. Equations can have one solution, infinitely many solutions, or no solution.

NUMBERS

$$\begin{aligned} 3x - 6 &= 3(x - 2) \\ 3x - 6 &= 3x - 6 \\ 3x - 3x - 6 &= 3x - 3x - 6 \\ -6 &= -6 \end{aligned}$$

The equation is true for all values of x . It is an identity. It has infinitely many solutions.

$$\begin{aligned} 3x - 2 &= 3x - 6 \\ 3x - 3x - 2 &= 3x - 3x - 6 \\ -2 &= -6 \end{aligned}$$

The equation is not true for any value of x . It has no solutions.

**Do You UNDERSTAND?**

- ESSENTIAL QUESTION** How do you create equations with a variable on both sides and use them to solve problems?
- Vocabulary** Why does it make sense to describe an equation that has infinitely many solutions as an *identity*?
- Error Analysis** Isabel says that the equation $x - 2 = -(x - 2)$ has no solution because a number can never be equal to its opposite. Explain the error Isabel made.
- Look For Relationships** You are solving an equation with a variable on each side. Does the side on which you choose to isolate the variable affect the solution? Why might you choose one side over the other?

Do You KNOW HOW?

Solve each equation.

- $5(2x + 6) = 8x + 48$
- $-3(8 + 3h) = 5h + 4$
- $2(y - 6) = 3(y - 4) - y$
- $8x - 4 = 2(4x - 4)$
- For how many games is the total cost of bowling equal for the two bowling establishments?

Family Bowling		
Cost (dollars)	Game	4.00
	Shoes	1.00
Knight Owl Bowling		
Cost (dollars)	Game	3.75
	Shoes	2.00

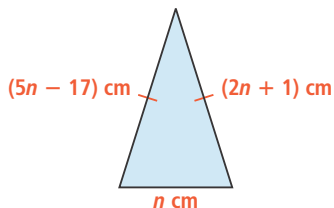


UNDERSTAND

10. **Reason** Do only equations with variables on both sides ever have no solution? Or can an equation with the variable on one side have no solution? Justify your answer.
11. **Generalize** How do you know whether an equation is an identity? How many solutions does an identity have? Explain.
12. **Error Analysis** Describe and correct any error a student may have made when solving the equation $0.15(y - 0.2) = 2 - 0.5(1 - y)$.

$$\begin{aligned}
 0.15(y - 0.2) &= 2 - 0.5(1 - y) \\
 0.15y - 0.3 &= 2 - 0.5 + 0.5y \\
 0.15y - 0.3 &= 1.5 + 0.5y \\
 (100)(0.15y - 0.3) &= 100(1.5 + 0.5y) \\
 15y - 30 &= 150 + 50y \\
 15y - 30 - 15y - 150 &= 150 + 50y \\
 &\quad - 15y - 150 \\
 -180 &= 35y \\
 -\frac{180}{35} &= y
 \end{aligned}$$

13. **Reason** When Nicky tried to solve an equation using properties of equality, she ended up with the equation $-3 = -3$. What equation might she have tried to solve? What is the solution of the equation?
14. **Mathematical Connections** The triangle shown is isosceles. Find the length of each side and the perimeter.



15. **Higher Order Thinking** The equation shown has a missing value.
- $$-2(2x - \square) + 1 = 17 - 4x$$
- For what missing value is the equation an identity?
 - For what missing value(s), if any, does the equation have exactly one solution?
 - For what missing value(s), if any, does the equation have no solution?

PRACTICE

Solve each equation. SEE EXAMPLES 1–3

- $5x - 4 = 4x$
- $7x = 8x + 12$
- $27 - 3x = 3x + 27$
- $34 - 2x = 7x$
- $5r - 7 = 2r + 14$
- $-x = 7x - 56$
- $5(n - 7) = 2(n + 14)$
- $6w - 33 = 3(4w - 5)$
- $3(x - 2) = 9x$
- $6(x + 5) = 3x$
- $\frac{4x + 6}{2} = \frac{3x - 15}{3}$
- $\frac{q + 1}{2} = \frac{q - 1}{3}$
- $2c + 3 = 2c + 3$
- $12b + 9 = 12b + 11$
- $x - 27 = -(27 - x)$
- $4(x + 9) = x + 9$
- $16(4 - 3m) = 96\left(-\frac{m}{2} + 1\right)$
- $6y - 8 = 2(3y - 4)$
- $5(5t + 1) = 25t - 7$
- $-3k + 4 = -2 - 6k$
- $\frac{1}{4}(2(x - 1) + 10) = x$
- $\frac{6x + 8}{2} - 4 = 3x$
- $3y = \frac{8 - 12y}{4} + 2$
- $0.25t = 0.25 - t$
- $0.625(x + 10) - 10 = 0$

Solve each problem. SEE EXAMPLE 4

41. Tavon has a \$50 gift card that loses \$2 for each 30-day period it is not used. He has a \$40 card that loses \$1.50 for each 30-day period it is not used.
- Write and solve an equation for the number of 30-day periods until the value of the gift cards will be equal.
 - What will the value of each card be when they have equal value?
42. A cereal box manufacturer changes the size of the box to increase the amount of cereal it contains. The equations $12 + 7.6n$ and $6 + 8n$, where n is the number of smaller boxes, are both representative of the amount of cereal that the new larger box contains. How many smaller boxes equal the same amount of cereal in the larger box?

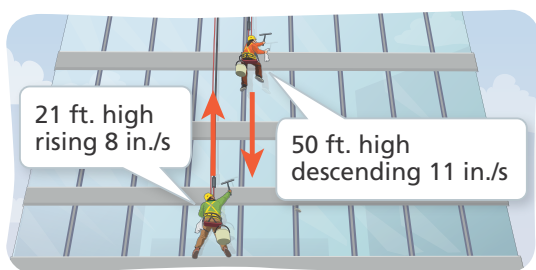




APPLY

- 43. Model With Mathematics** Arthur wants to buy an item that costs p dollars before tax. Using a 6% sales tax rate, write two different expressions that represent the price of the item after tax. Show that the two expressions are equal.

- 44. Model With Mathematics** Two window washers start at the heights shown. One is rising, the other is descending. How long does it take for the two window washers to reach the same height? Explain.



- 45. Construct Arguments** Jamie will choose between two catering companies for an upcoming party. Company A charges a set-up fee of \$500 plus \$25 for each guest. Company B charges a set-up fee of \$200 plus \$30 per guest.

- Write expressions that you can use to determine the amount each company charges for g guests.
- Jamie learns that the \$500 set-up fee for Company A includes payment for 20 guests. The \$25 per guest charge is for every guest over the first 20. If there will be 50 guests, which company will cost the least? Explain.

- 46. Construct Arguments** A one-year prepaid membership at Gym A costs \$250 plus \$19 per month for the second year. A one-year prepaid membership at Gym B costs \$195 plus \$24 per month for the second year. Leah says the cost for both gym memberships will be the same after the 11th month of the second year. Do you agree? Explain.

- 47. Model With Mathematics** A red balloon is 40 feet above the ground and rising at 2 ft/s. At the same time, a blue balloon is at 60 feet above the ground and descending at 3 ft/s. What will the height of the balloons be when they are the same height above the ground?



ASSESSMENT PRACTICE

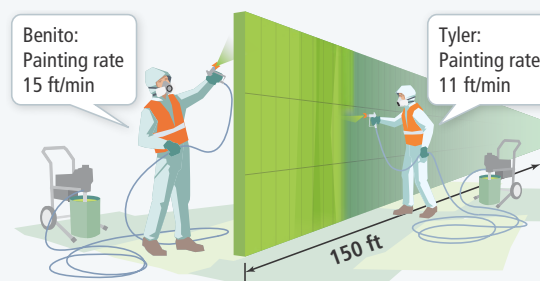
- 48.** Which equations have no solution? Select all that apply.

- $x - 9 = 2(x - 3) + 12$
- $5(-2x + 7) + 3 = -10x + 38$
- $\frac{1}{2}(6x - 4) = 3(x - 2)$
- $0.01x + 0.001 = \frac{1}{100}(x + 10)$
- $3(x + 2) + 1 = x + 2(4 + x)$

- 49. SAT/ACT** Which equation is an identity?

- $\frac{9x}{15} + 27 = \frac{9x}{15} + \frac{27}{15}$
- $3\left(\frac{x}{2} + 16\right) - 16 = \frac{3}{2}x$
- $-4(3 - 2x) = -12 - 8x$
- $-5\left(\frac{x}{15} - 16\right) - 30 = 50 - \frac{1}{3}x$
- $36\left(\frac{3}{4}x - 2\right) + 72 = -72 + 27x$

- 50. Performance Task** Benito and Tyler are painting opposite sides of the same fence. Tyler has already painted $19\frac{1}{2}$ feet of his side of the fence when Benito starts painting.



Part A How long will it take for the two sides of the fence to have an equal number of feet painted? How many feet will be painted on Benito's side of the fence when the two sides have an equal number of feet painted?

Part B Tyler claims that because he started painting first, he will finish painting his side of the fence before Benito finishes painting his side. Is this true? Explain.

Part C The painter who finishes first gets to rest while the other painter finishes. How long will the painter who finishes first get to rest? Explain.

11. If an equation is an identity then it simplifies to $0 = 0$, and there are an infinite number of solutions; Sample reasoning: substituting any number for all occurrences of the variable results in a true equation.
13. Answers may vary. Sample answer: $x - 3 = x - 3$; It is an identity, so it has an infinite amount of solutions.
15. a. 8.
b. no values
c. any value except 8
17. -12
19. $\frac{34}{9}$
21. 7
23. -3
25. -10
27. -5
29. no solution
31. -9
33. identity
35. -2
37. identity
39. 0.2
41. a. $50 - 2d = 40 - 1.5d$; 20 30-day periods
b. \$10
43. Answers may vary. Sample answer: $p + 0.06p$ and $1.06p$;
 $p + 0.06p = (1 + 0.06)p = 1.06p$

- 45.** **a.** Company A: $500 + 25g$. Company B: $200 + 30g$
- b.** Company A; Cost for Company A is $500 + 25(30) = \$1,250$; Cost for Company B is $200 + 30(50) = \$1,700$. So, A costs less.
- 47.** 48 ft
- 49.** D